V Semester B.A./B.Sc. Examination, November/December 2018
(CBCS) (2016 – 17 and Onwards) (Semester Scheme)
(Fresh + Repeaters)
MATHEMATICS – VI

Time : 3 Hours Max. Marks : 70

Instruction : *Answer all questions.*

PART – A

1. Answer **any five** questions. **(5×2 =10)**

a) Write Euler's equation when f is independent of y.

b) Find the differential equation of the functional $I = \int_{x_1}^{x_2} \left[y^2 - (y')^2 + 2ye^x \right] dx$.

c) Write the Euler's equation.

d) Evaluate $\int_C (3x + y)\,dx + (2y - x)\,dy$ along y = x from (0, 0) to (10,10).

e) Evaluate $\int_0^{\pi/2} \int_0^{a\cos\theta} r^2\,dr\,d\theta$.

f) Evaluate $\int_0^1 \int_0^2 \int_1^2 xyz\,dx\,dy\,dz$

g) Find the area of the circle $x^2 + y^2 = a^2$ by double integration.

h) State Stoke's theorem.

PART – B

Answer **two full** questions. **(2×10 =20)**

2. a) Find the extremal of the functional $I = \int_0^{\pi/2} \left[y^2 - (y')^2 - 2y\sin x \right] dx$ under the end conditions $y(0) = y(\pi/2) = 0$.

b) Define Geodesic. Prove that geodesic on a plane is a straight line.

OR

3. a) If a cable hangs freely under gravity from two fixed points then show that the shape of cable is a catenary.

b) Solve the variational problem $\delta\int_0^{\pi/2}\left[y^2-(y')^2\right]dx=0$ under the condition $y(0)=0,\ y\left(\pi/2\right)=2$.

4. a) Prove that catenary is the curve which when rotated about a line generates a surface of minimum area.

b) Find the extremal of the functional $\int_{x_1}^{x_2}\left[12xy+(y')^2\right]dx$.

OR

5. a) Find the extremal of the functional $\int_0^1\left[x+y+(y')^2\right]dx=0$ under the conditions $y(0) = 1$ and $y(1) = 2$.

b) Find the extremal of the functional $\int_0^1\left[(y')^2+x^2\right]dx$ subject to the constraint $\int_0^1 y\,dx=2$ and having end conditions $y(0) = 0$ and $y(1) = 1$.

PART – C

Answer **two full** questions. **(2×10=20)**

6. a) Evaluate $\int_C(x+y+z)\,ds$ where C is line joining the points (1, 2, 3) and (4, 5, 6) whose equations are $x = 3t + 1$, $y = 3t + 2$; $z = 3t + 3$.

b) Evaluate $\iint_R xy(x+y)\,dx\,dy$ over the region R bounded between the parabola $y=x^2$ and the line $y = x$.

OR

7. a) Change the order of integration in $\int_0^a\int_0^{2\sqrt{ax}}x^2\,dx\,dy$ and hence evaluate.

b) Evaluate $\iint_A\sqrt{4x^2-y^2}\,dx\,dy$ where A is the area bounded by the lines $y = 0$, $y = x$ and $x = 1$.

8. a) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz\,dxdydz$

b) Change into polar coordinates and evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}\,dx\,dy$.

OR

9. a) Find the volume of the sphere $x^2 + y^2 + z^2 = a^2$ using triple integration.

b) Evaluate $\iiint xyz\,dxdydz$ over the positive octant of the sphere $x^2 + y^2 + z^2 = a^2$ by changing it to spherical polar coordinates.

PART – D

Answer **two full** questions. **(2×10 =20)**

10. a) State and prove Green's theorem.

b) Using divergence theorem, evaluate $\iint_S \left(x\hat{i} + y\hat{j} + z^2\hat{k}\right).\hat{n}ds$ where S is the closed surface bounded by the cone $x^2 + y^2 = z^2$ and the plane $z = 1$.

OR

11. a) By using divergence theorem, evaluate $\iint_S \vec{F}.\hat{n}ds$ where $\vec{F} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$ and S is the surface enclosing the region for which $x^2 + y^2 \le 4$ and $0 \le z \le 3$.

b) Evaluate $\iint \text{curl}\,\vec{F}.\hat{n}\,ds$ by Stoke's theorem if $\vec{F} = (y - z + 2)\hat{i} + (yz + 4)\hat{j} - xz\hat{k}$ and S is the surface of the cube $0 \le x \le 2, 0 \le y \le 2, 0 \le z \le 2$.

12. a) Using Green's theorem evaluate for the scalar line integral of $\vec{F} = (x^2 - y^2)\hat{i} + 2xy\hat{j}$ over the rectangular region bounded by the lines $x = 0, y = 0 ; x = a; y = b$.

b) Using the divergence theorem evaluate $\iint_S \vec{F}.\hat{n}ds$ where $\vec{F} = (x^2 - yz)\,\hat{i} + (y^2 - zx)\,\hat{j} + (z^2 - xy)\,\hat{k}$ over the rectangular parallelopiped $0 \le x \le a,\ 0 \le y \le b,\ 0 \le z \le c$.

OR

13\. a) Using Green's theorem evaluate $\int_C (xy + y^2)\,dx + x^2dy$ where C is the closed curve bounded by $y = x$ and $y = x^2$.

b) Evaluate by Stoke's theorem $\oint_C yz\,dx + zx\,dy + xy\,dz$. where C is the curve $x^2 + y^2 = 1;\ z = y^2$.

BMSCW